DOCUMENT RESUME

ED 389 543. SE 057 186

AUTHOR Doerr, Helen M.

TITLE Evidence and Proof: Explaining Vector Relationships.

PUB DATE Oct 95

NOTE 8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.

PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.

DESCRIPTORS *Geometry; High Schools; High School Students; Mathematics Instruction; *Proof (Mathematics); Science Instruction; *Scientific Methodology; *Secondary School Mathematics; Small Group Instruction

IDENTIFIERS *Representations (Mathematics)

ABSTRACT

This paper addresses a central issue in secondary school geometry, namely the role of proof. In an integrated mathematics and science curriculum, the role of proof as a process of conjecturing, explaining, and justifying within a small-group setting is analyzed. Particular attention is given to the students' use of empirical evidence and algebraic representations. Each class session of this unit was videotaped, and during small group work, the focus group was videotaped. Written work and computer work done by the group were made available to the researcher for analysis. Extensive field notes were taken by the researcher during class sessions and videotapes of class sessions were reviewed and transcribed for more detailed analysis. (Author/MKR)

Evidence and Proof: Explaining Vector Relationships

Helen M. Doerr

A Paper Presented at the Seventeenth Annual Meeting for the Psychology of Mathematics Education (North American Chapter)

October 21-24, 1995

EVIDENCE AND PROOF: EXPLAINING VECTOR RELATIONSHIPS

Helen M. Doerr, Syracuse University

This paper addresses a central issue in secondary school geometry, namely the role of proof. In an integrated mathematics and science curriculum, the role of proof as a process of conjecturing, explaining and justifying within a small group setting is analyzed. Particular attention is given to the students' use of empirical evidence and algebraic representations.

The role of proof in high school mathematics continues to be a topic of debate among educators. Such debate is frequently centered on the development of formal axiomatic systems in the context of teaching and learning geometry. Recent work with alternatives to axiomatic approaches to geometry has led to research on students' learning with computer-based construction programs such as the Geometric Supposer and the Geometer's Sketchpad. These environments have been shown to be effective in supporting students' exploration of geometry and in the making of conjectures and explaining and justifying their ideas (Chazan, 1993). However, many of these studies do not include pencil and paper construction, mechanical devices, or physical experimentation, but rather begin with the use and modification of representations in the computational medium. In this study, we examine a process of conjecturing, explaining, and justifying that begins with physical experimentation and then uses a multi-representational analysis tool with student-generated diagrams to support the students' development of a convincing argument about the relationships among multiple force vectors acting on an object at rest. This study grounds the explanatory role for proof, suggested by Hanna (1990), in a physical experiment with forces.

Description of Study

This paper will address the development of a convincing argument by one small group of students for the relationships among the forces acting on an object at rest on an inclined plane. This study was part of a larger research project on an integrated modeling approach for building student understanding of the concepts of force and motion and enhancing problem-solving skills. In this larger study, we examined a modeling process which integrated three components: the action of building a model from physical phenomena, the use of simulation and multiple representations, and the analysis, refinement and validation of potential solutions. In this paper, we present an analysis of the development of a geometric argument for vector relationships, which includes the formation of multiple conjectures, qualitative reasoning about those conjectures, and the refinement and validation of the conjectures.

Data Sources and Analysis

Each class session of this unit was videotaped, and during small group work, the focus group of this study was videotaped. Written work and computer work done by the group were made available to the researcher for analysis. Extensive field notes were taken by the researcher during the class sessions. The videotapes of class sessions were reviewed and transcribed for more detailed analysis.

Description of the Curricular Activities

The unit began with a simple physical experiment: an object was held suspended just above an inclined plane so that the plane served as a reference frame for the changing angle of inclination (see Figure 1). Using two spring scales, some rope and an object of known weight, the force parallel to the plane and the force perpendicular to the plane were measured for various settings of the angle theta between zero and 90 degrees. The teachers then posed a very open-ended inquiry to the class: how do these forces relate to the weight of the object, the angle of inclination, and/or to each other? In an earlier unit, the students had established that a force acting at angle can be thought of as having a vertical and a horizontal component and that these components are related trigonometrically.

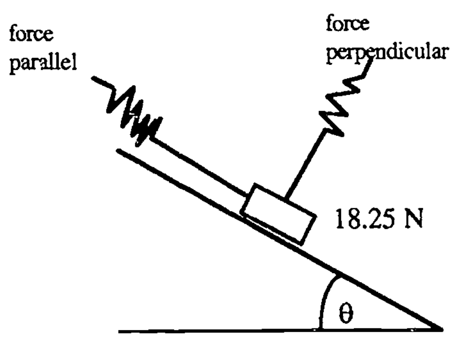

Figure 1. Forces Acting Along an Incline

The students were now faced with the problem of developing one or more conjectures about the relationships between and among the parallel and perpendicular forces, the weight of the object, and the angle of inclination of the plane. Then, in light of these conjectures and the evidence provided by the experimental data for the range of cases between zero and 90 degrees, the students were asked to convince themselves of the validity of one or more of these conjectures.

Results

The focus group of this study began by entering the experimental data into a Function Probe table (see Figure 2). From the table window, Alycia observed that the forces do not add up to a constant and that the forces change at the same rate. She appeared to be observing the symmetry of the covariation of the data for the parallel and perpendicular forces. The students quickly decided to graph the data, placing angle on the x-axis and forces on the y-axis. After graphing the first relationship, Paul observed that the second is symmetrical to the first; Alycia and Jenny confirmed the equality at 45 degrees. After these preliminary observations, they identified that their task was to find a relationship among the variables. At

θ	p	P
angle	parallel	perp.
0.00	0 00	18.25
10.00	3.75	18.00
20.00	7 25	17.25
30.00	9.25	15.75
40.00	11.00	14.00
45.00	12 75	13.00
50.00	13.75	12.00
60.00	15 75	10.25
72.00	17.50	6.50
82.00	18.10	4.50
90.00	18 25	0.00

Figure 2. Parallel and Perpendicular Forces along an Inclined Plan

this point, they had in front of them, in the graph window, a relationship between the angle of inclination, and the force perpendicular and the force parallel. But nonetheless, for the students, it did *not* answer the question of what is the relationship? The inquiry which follows suggests that the students are looking for an explanation as to why this relationship holds. Paul suggested that they leave the graph window and return to the table, but then Jenny directed the group to the geometry of the situation.

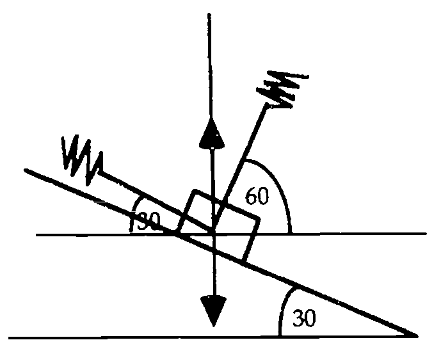

Figure 3. *Paul's Force Diagram*

From their drawings (see Figure 3), they began to clarify the meaning of the force data that they had collected. They began to analyze the role of the vertical and horizontal components of the forces. They continued to explore the symmetry of the geometry: they noted the equality of the forces at 45 degrees, that the forces are opposite at 30 and 60 degrees, and that there are complementary angles within the force diagram. These complementary angles later turned out to be crucial to their final argument. They clearly expressed qualitatively the relationship between the weight of the object and the vertical components of the parallel and perpendicular forces. They then tried to establish an equation for that relationship in the table window:

P: So the vertical component of this [parallel] force is equal to 4.625.

A: And minus that from 18.25 and get the other vertical component.

P: Yeah. So we can make an equation.

A: Let's make another column for vertical.

This reasoning and analysis built on their earlier understanding of the vertical and horizontal components of a force at an angle from the previous sub-unit. They were able to calculate the magnitude of the vertical components of the forces. They knew that the horizontal components could also be computed and that they must be equal. However, their efforts to express the relationship of the weight to the vertical components algebraically in the table window were unsuccessful.

At this point, Jenny restated her earlier lack of confidence in this strategy and suggested that the path they were heading down was one that would lead to a series of relationships (that might in fact be circular) rather than just one:

A: That's what? Round about?

J: Are we gonna, yeah, are we, yeah, totally! I mean that's not going to give us a, that's going to give us a series of relationships. Rather than looking for just one.

P: It's gonna give us, right, well, we can use that whole thing as the relationship between that and that. [the parallel and perpendicular forces]

J: Okay, uh huh.

The group took a short break, but Paul continued to work, going back to the force diagram, repeating his calculations, and attempting to create a relationship in the table window of Function Probe. By the time Jenny and Alycia returned, Paul was not able to create an algebraic relationship to give him the vertical component that he was looking for. At this point, the teacher provided some general instruction to the whole class and gave the groups another data set, so that they could verify that whatever relationships they developed would hold for alternative scenarios. The teacher then joined Jenny, Alycia and Paul. They explained their thinking about the component forces and showed Dave their graphs. Dave focused their attention on the original graph and pointed out that they could read the relationship between the angle and the forces (the original question) directly from the graph. Both Jenny and Paul suggested for the first time that these might be trigonometric curves. They then algebraically fit a sine and a cosine curve to their data and, at the very close of class, established that the magnitude of the perpendicular force is given by the weight of the object times the cosine of the angle of incline, and similarly for the parallel force. As class ended, the group did not bring any closure to the central idea that *they* were working on, namely, that the sum of vertical components of the perpendicular and parallel force must equal the weight of the object.

Class began the next day with a whole class discussion recapping that the relationships which the small groups had found between the parallel and the perpendicular forces, the weight of the object and the angle were given by the equations F^ = Fw * cosq and F||= Fw * sinq and were represented graphically by a pair of curves intersecting at 45 degrees. The teacher pointed out to the students that these relationships were created on the basis of fitting a curve to empirical data and that they didn't have a visual, pictorial argument to support the fact that they came up with sines and cosines. He directed the students to develop a convincing argument for the relationships on the inclined plane based on the geometry of the force diagram. The students moved to an analysis of the geometry in front of them. They had found the two equal angles (see Figure 4) yesterday and now they quickly put that together in a geometric argument to support the empirical curve fit that had been done:

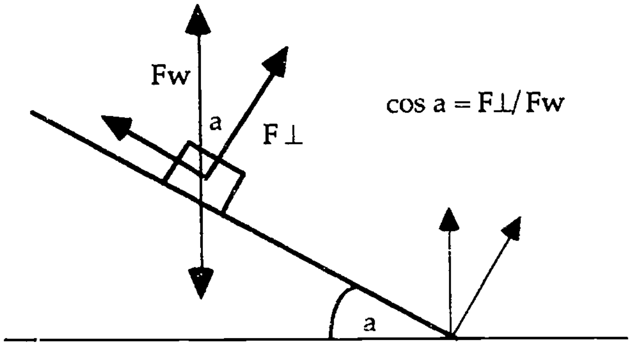

Figure 4. Alycia's Force Diagram

P: No wait, you guys, guys, guys, the reason why this, the, the cosine of this angle, which is also that angle, um (pause).

A: The cosine of this angle is this angle?

P: Yeah! No! The cosine of this angle is the same as the cosine of that angle. It's the same angle.

A: Yeah, hm, hm.

D: Is that the same angle?

P: The cosine of, yeah.

A: Yeah, because it's this minus 90. And these two are the same.

Alycia proceeded to write a two-column proof that argued that if two angles have the same complement then the angles must be equal. They then were convinced that the cosine relationship that they found empirically must also hold from the geometry of the force diagram:

A: So, if these two angles are equal, then the forces...

P: Then, right, then it makes sense that the cosine of this angle times that would equal that force. Cause this...

A: You can just draw this angle down here. It can be anywhere.

P: This force. Right. That force can be anywhere. Cause...

A: I just moved it down, so that it's easier to see that they're equal.

P: Right.

D: Hm, hm.

A: And so, of course, two angles that are equal have the same cosine.

They proceeded to make the analogous argument for the parallel force.

Discussion

The focus group had built an understanding of the force relationships involved for an object at rest on a ramp through a process that involved collecting empirical data, graphing the data, examining the table values, constructing a force diagram and then analyzing the vertical and horizontal components of the forces. Their qualitative understanding of the relationship of the vertical components included both the additive relations of the vertical components to the weight of the object and the symmetry between those components. Their analysis showed both an understanding of the relationship between the vertical components of the parallel and perpendicular forces and an argument as to why those components must equal the weight of the object. However, this analysis did not include a symbolic, tabular or graphical representation of the relationship. Directed by the teacher back to the original graph of the empirical data, the students left their model and generated an algebraic relationship from a curve fitted to the data. The students ultimately returned to the geometry of the force diagram, arguing that the empirical trigonometric relationships from the curve fitting made sense in terms of right triangle trigonometry. Thus, while they never returned to their earlier attempts at an algebraic analysis of the vertical and horizontal components of the parallel and perpendicular forces, those attempts led them to a clear and compelling analysis of the overall scenario.

References

Chazan, D. (1993). High school geometry students' justification for their views of empirical evidence and mathematical proof. *Educational Studies in Mathematics, 24*, 359-387.

Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange, 21*(1), 6-13.